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Shape Optimization of Stratosphere Airship

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I. Introduction

PRESENTLY, there is a strong interest in developing unmanned stratospheric airship platforms to be utilized as telecommunication relays, for environmental monitoring or surveillance purposes. Because of the large surface area of the hull, an airship shows high drag, which roughly increases with the square of the airspeed. The required propulsion power is proportional to the third power of the airspeed. Therefore, it is of essential interest to minimize the aerodynamic drag and to maximize the propulsion efficiency. The propulsive power required depends mainly on the aerodynamic drag of the airship hull, which accounts for about $\frac{2}{3}$ of the total drag. Even a small reduction in hull drag can result in a significant saving of fuel, which in turn will lead to a greater payload capacity or an increased range of the airship. During the aerodynamic design of an airship, it is therefore especially important to find a drag-minimized envelope for the intended range of missions.

The investigations on the shape optimization of airship were conducted by Th. Lutz et al.[1] and Nejati et al.[2] A source distribution on the body axis was chosen to model the body contour and the corresponding inviscid flowfield, with the source strengths and the lengths of respective segments being used as design variables for the optimization process. Boundary-layer calculation is performed by means of a proved integral method for attached laminar or turbulent boundary layers. To determine the transition location, Lutz used a semi-empirical method based on linear stability theory (e^n-method), Nejati applied forced transition criterion and concluded that the e^n-method is desirable for determining the transition location. A commercial optimizer as well as an evolution strategy with covariance matrix adaption of the mutation distribution were applied as optimization algorithm in Lutz' paper. Nejati used the genetic algorithm as the optimization algorithm and found that genetic algorithm is a powerful method for such a multidimensional, multimodel, and nonlinear objective function.

In this Note, we developed a new method for shape optimization of airship body in terms of the previous investigations of shape optimization.[1–11] The airship geometry is expressed analytically as a polynomial function of eight parameters. The inviscid flow is computed by distribution of source on airship surface, which provides the pressure and velocity of airship's surface. Through the computation results of drag coefficient with different theory, Pater[10] concluded that the sensitivity of the Squire–Young formula of drag coefficient with respect to integral parameters at the trailing edge is not well. Different theories can get different integral parameters. However, the drag coefficient is almost equal by putting them into the Squire–Young formula. Therefore, if we only need the value of drag coefficient it is enough to compute drag coefficient using the thin boundary-layer theory. So, in this Note the boundary layer is computed using an integral formulation of thin boundary-layer theory: the laminar part of the flow is computed with Thwaites' model, and the turbulent part is solved with Head's model. A e^n-type amplification formulation is used to locate the transition area. The drag coefficient is computed using the Squire–Young formula. Optimization problems are solved using a hybrid genetic algorithm composed of genetic algorithm and Nelder–Mead simplex search method. Using a one-way coupled inviscid boundary-layer model and a hybrid genetic algorithm, shape optimization of airship in incompressible flow at zero incidence was performed.

II. Aerodynamic Solver

A. Inviscid Flow Model

The inviscid part of the flow can be solved in two ways: a finite difference discretization of the steady Euler equation and a panel method. The inviscid flow solver provides the tangential velocity distribution on the airship's surface. The pressure distribution is then computed from the velocity field using the Bernoulli equation.

For an irrotational flow, the velocity is the gradient of a quantity called the velocity potential:

$$V = (u, v, w) = \nabla\phi$$

Substituting this into the continuity equation for an inviscid incompressible flow leads to the following:

Laplace equation:

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0$$

On the airship's surface S, the external Neumann boundary condition must be satisfied:

$$\frac{\partial\phi}{\partial n} = V_n$$

where V_n can be related to integral boundary-layer quantities through the transpiration velocity model. In our case, the effect of the boundary layer is neglected, and V_n is simply set to zero, which produces the classical zero normal velocity condition.

The two most common approaches to solve potential flow around axisymmetric bodies are based on representing the body by a distribution of singularities, either along its axis or on its surface. The axial singularity approach is simpler and computationally much more efficient, whereas the surface singularity approach is more accurate for bodies with sudden changes in curvature and can handle nonaxisymmetric shapes. During the course of optimization, the geometry with sudden changes in curvature can be met, and so we use the surface singularity approach.

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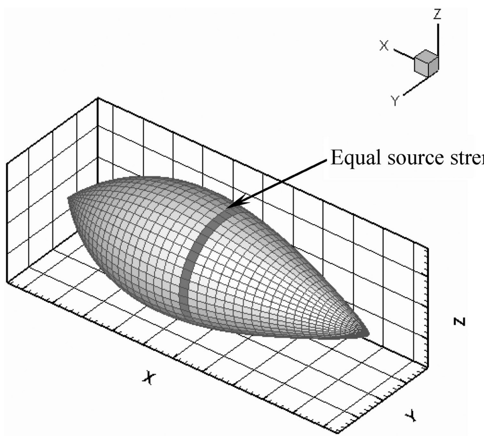

Fig. 1 Mesh and equal source strength in cross section of airship body.

In this Note, we use an improved panel method from the Hess–Smith method[12] to provide the pressure and velocity of airship's surface. The mesh of airship is generated along the x direction of body coordinates and circumference direction. The airship body along the x direction is divided into N parts. The circumference is divided into M parts. It is shown in Fig. 1. Thus, the source strength distributed on the panel in cross section of airship is equal because of the axial symmetry of airship in incompressible flow at zero incidence. The number of unknown source strength is decreased from $M \times N$ to N. Thus, the computation is as efficient as that of axial singularity approach. Moreover, this method is more accurate than axial singularity approach.

B. Boundary-Layer Model

Because of the axisymmetry of airship at zero incidence, the computation of the boundary layer of the airship can be transformed into a two-dimensional boundary layer by Mangler transformation.[13] The two-dimensional boundary-layer formulation consists of a model for the laminar part of the flow, a transition criterion, and a model for the turbulent part of the flow. The flow models rely on one or two differential equations derived from the integration of the Falker–Skan equations across the boundary-layer thickness and on additional semi-empirical equations, which close the system. Before using them in the boundary-layer model, the external velocity distribution and the coordinates of the airship must be made dimensionless. The external velocity is thus divided by the far-field flow velocity, and the airship point coordinates are divided by the airship length.

1. *Thwaites' Model for the Laminar Part*

The Thwaites' model relies on the integral momentum equation. It can be expressed as follows:

$$ReU_e \frac{d\theta^2}{dx} = 2[L - (2 + H)\lambda] \approx 0.45 - 6\lambda \tag{1}$$

where θ is the momentum thickness, H is the shape factor, U_e is the dimensionless tangential velocity distribution on the airship's surface, $L = Re\theta U_e C_f/2$, $\lambda = Re\theta^2 dU_e/dx$, $Re = U_\infty l/\nu$ is the Reynolds number, l is the length of airship, ν is the kinematic viscosity of atmosphere, and U_∞ is far-field flow velocity.

The definition of λ is substituted in Eq. (1), and then we obtain

$$ReU_e \frac{d\theta^2}{dx} = 2[L - (2 + H)\lambda] = 0.45 - 9Re\theta^2 \frac{dU_e}{dx}$$

$$\Rightarrow Re\frac{d}{dx}\left(\theta^2 U_e^6\right) = 0.45U_e^5$$

The value of θ at the stagnation point is known:

$$\theta|_{x=0} = \sqrt{\frac{0.075}{Re(dU_e/dx)|_{x=0}}}$$

Starting from that, the integration is performed as follows:

$$Re\left(\theta^2 U_e^6\right)\Big|_{i-1}^{i} = 0.45 \int_{x_{i-1}}^{x_i} U_e^5 \, dx$$

Once θ is known, λ can be calculated. The shape factor H and the skin-friction coefficient C_f are then computed from semi-empirical formulas given by Cebeci and Bradshaw in Ref. 14.

As all of the one-equation methods, the Thwaites method cannot represent separated flows because it uniquely ties the shape parameter to the local pressure gradient, which is, in fact, a nonunique relationship in separating flows. Therefore, for some cases laminar separation (detected by the vanishing of C_f) is obtained before transition. After that, laminar separation is considered as a trigger for transition, and the computation is carried on with the turbulent flow model.

2. *The e^n Criterion of Locating Transition*

In this Note, the laminar-to-turbulent transition is determined by a semi-empirical e^n method based on linear stability theory. The correlation of a large number of wind-tunnel data and flight transition experiments with linear boundary-layer stability calculations has made the e^n method a consistent transition prediction method. Using the Falker–Skan profile family, the spatial amplification curve envelopes can be related to the local boundary-layer parameters. The procedure is described in Ref. 4. The envelopes are then approximated by straight lines:

$$\tilde{n} = \frac{d\tilde{n}}{dRe_\theta}(H)\left[Re_\theta - Re_{\theta_0}\right] \tag{2}$$

where $\tilde{n}$ is the logarithm of the maximum amplification ratio. The slope $d\tilde{n}/dRe_\theta$ and the critical Reynolds number Re_{θ_0} are expressed by the following empirical formulas:

$$\frac{d\tilde{n}}{dRe_\theta} = 0.01\{[2.4H - 3.7 + 2.5\tanh(1.5H - 4.65)]^2 + 0.25\}^{\frac{1}{2}}$$

$$\log_{10}\left(Re_{\theta_0}\right) = \left(\frac{1.415}{H-1} - 0.489\right)\tanh\left(\frac{20}{H-1} - 12.9\right) + \frac{3.295}{H-1} + 0.44$$

For simple cases such as symmetric airship at zero angle of attack, Re_θ is uniquely related to the streamwise coordinate x, and Eq. (2) immediately gives the amplitude ratio $\tilde{n}$ as a unique function of x. Transition is assumed to occur where $\tilde{n}(x) = 9$.

3. *Turbulent Boundary Layer*

Head's model is a reasonably accurate and especially fast method for the computation of turbulent boundary layer. The model uses an integral momentum equation and some semi-empirical relations to close the system. The method has been expressed as follows:

$$\frac{d\theta}{dx} = -\frac{\theta}{U_e}(2 + H)\frac{dU_e}{dx} + \frac{1}{2}C_f$$

$$\frac{dH_1}{dx} = -H_1\left(\frac{1}{U_e}\frac{dU_e}{dx} + \frac{1}{\theta}\frac{d\theta}{dx}\right) + \frac{0.0306}{\theta}(H_1 - 3)^{-0.6169}$$

$$H_1 = k(H) = \begin{cases} 3.3 + 0.8234(H - 1.1)^{-1.287}, & H \le 1.6 \\ 3.3 + 1.5501(H - 0.6778)^{-3.064}, & H > 1.6 \end{cases}$$

$$C_f = 0.246 \cdot (10^{-0.678H}) Re_\theta^{-0.268}$$

Turbulent separation really corresponds to the case when $H1$ approaches 3.3 and H increases rapidly. A typical value for H at separation often presented in the Note is $H = 2.4$. This can be used as a separation criterion, because H increases very rapidly close to separation anyway. In our case, we use the criterion $H1 = 3.3$.

4. Computation of the Drag Coefficient

The Squire–Young formula[15] provides a means of predicting the profile drag by relating the momentum defect far downstream to the values of the flowfield given at the trailing edge. Given the dimensionless momentum thickness θ, shape factor H, and dimensionless velocity U_e at the trailing edge on generatrix, the volumetric drag coefficient is found from

$$C_{\text{dv}} = \frac{\text{Drag}}{\frac{1}{2}\rho U_\infty^2 V^{\frac{2}{3}}} = \frac{4\pi}{V^{\frac{2}{3}}} r_0 \theta (U_e)^{(H+5)/2}|_{\text{TE}}$$

where Drag is the drag of airship, ρ is the density of atmosphere, and V is the volume of airship body.

This formula gives directly the total profile drag (i.e., the sum of pressure drag and friction drag) as a function of the values of r_0, $U_e\theta$, and H at the trailing edge. It is thus particularly well suited for one-way coupled methods, in which the computation of the pressure drag is not possible because the inviscid flow is not influenced by the effect of the boundary layer.

5. Coupling of Inviscid Flow Model and Boundary-Layer Model

Two different approaches are possible for the coupling of inviscid boundary-layer flows: two-way coupled computation and one-way coupled computation.

Two-way coupled computation. The solution begins with the inviscid flow problem, which produces the velocity field. These data are then fed into the boundary-layer model, which results in the local wall-friction coefficient and the displacement thickness. Then a second iteration is performed, now with modified surface geometry. This modification can be obtained by displacing the body panels according to the local displacement thickness, and the procedure is reiterated until a converged solution is obtained. Another way to account for the displacement effects is to modify the boundary condition instead of the geometry. A study of such iterative methods can be found in Ref. 5, where it appears that convergence is not easy to obtain.

One-way coupled computation. In this case the effect of the boundary-layer thickness is neglected. One single iteration is performed: the external tangential velocity is computed by the inviscid model with the condition $V_n = 0$ on the airship's surface and then fed into the boundary-layer model. The drag coefficient is obtained using the Squire–Young formula, which in effect computes the momentum deficit. It is a function of some of the boundary-layer results (momentum thickness and shape factor) at the trailing edge. Compared to two-way coupled computations, the time of computation consumed is much less, but the drag is not accuracy. However the sensitivity of the drag with respect to airship shape or flow parameters is well reproduced.[6] It is enough for the shape optimization of airship. This method is used in the present Note.

III. Airship Body Geometry

There are several approaches such as curve interpolation, polynomial fitting, or linear superposition of analysis functions that can express the airship geometry. In this Note, the airship geometry is expressed analytically as a polynomial function of eight parameters.[7,8] The airship body is divided into three sections: forebody section, midbody section, and aftbody section. Eight parameters were chosen as design variables to represent the body geometry.

Forebody section:

$$0 \le z \le z_m$$

$$r_0(x)/l = (1/2f_r)[r_n F_1(x) + k_1 F_2(x) + G(x)]^{\frac{1}{2}}$$

$$x = z/z_m \qquad F_1(x) = 2x(x-1)^2$$

$$F_2(x) = -x^2(x-1)^3 \qquad G(x) = x^2(3x^2 - 8x + 6)$$

Midbody section:

$$z_m \le z \le z_i$$

$$r_0(x)/l = (1/2f_r)\{r_i + (1-r_i)[k_{1m}F_1(x) + S_i F_2(x) + G(x)]\}$$

$$x = (z_i - z)/(z_i - z_m) \qquad F_1(x) = -\tfrac{1}{2}x^3(x-1)^2$$

$$F_2(x) = x - x^3(3x^2 - 8x + 6) \qquad G(x) = x^3(6x^2 - 15x + 10)$$

$$k_{1m} = k_1[(z_i/z_m - 1)^2/(1 - r_i)]$$

Aftbody section:

$$z_i \le z \le l \qquad \frac{r_0(x)}{l} = \frac{r_i}{2f_r}\left[1 + \left(\frac{t}{r_i} - 1\right)F_1(x) + S_m F_2(x)\right]$$

$$x = \frac{(l-z)}{(l-z_i)} \qquad F_1(x) = 1 - x^3(6x^2 - 15x + 10)$$

$$F_2(x) = -x^3(3x^2 - 7x + 4) \qquad S_m = \frac{(1-r_i)(1-x_i)}{(x_i - x_m)} \cdot S_i$$

where r_n is dimensionless curvature radius of forehead, f_r is fineness ratio of airship body $= l/D$, x_m is the dimensionless x value of maximum radius of airship body $= z_m/l$, k_1 is the dimensionless curvature in maximum radius, x_i is the dimensionless x value of inflexion $= z_i/l$, r_i is the dimensionless radius of inflexion $= (2f_r)r_0(z_i)/l$, S_i is the geometry slope ratio of inflexion, and t is the dimensionless radius of tail $= (2f_r)T/l$.

Because of the closed contour of airship body, we set the parameters $x_i = 1.0$, and $r_i = t = 0.001$.

IV. Shape Optimization of Airship Body

The design volume Reynolds number of high-altitude stratospheric platforms can be in the order of $10^7 \le Re_v \le 3 \times 10^7$. For the shape optimizations, it became obvious that one-point optimizations for a single Reynolds number lead to bodies that are inconvenient or even unusable outside of their point.[9] For this reason, the shape optimizations in this Note were performed for N different equal distance points in a Reynolds-number regime.

Definition of the optimization problems as follows:

$$\min \quad J_0 = \left(\sum_{i=1}^{N} \omega_i C_{\text{dvi}}\right) \Big/ N$$

Subject to:

Eight parameter ranges and given volume of airship body

where $\omega_i (i = 1, \ldots, N)$ are weighting factors, $C_{\text{dvi}} (i = 1, \ldots, N)$ are volumetric drag coefficients, and Re_v is the volumetric Reynolds number. The volumetric Reynolds number are being evaluated:

$$Re_v = U_\infty V^{\frac{1}{3}} \big/ \nu$$

V. Solving the Optimization Problems

Optimization problems are solved using a hybrid genetic algorithm[16,17] composed of the genetic algorithm and Nelder–Mead simplex search method. Genetic algorithm aims at the global search and simplex method for intensification within these possible optimal results. This method can efficiently improve the ability of searching for optimal result. Figure 2 show the pseudocode of the hybrid method.

VI. Computational Study

Because of the requirements such as inner component distribution, stability and so on, there will be some constraints to the shape

Table 1 Results of shape optimization of airship body

Parameters	$f_r \in [1.5, 8.0]$
r_n	0.4564368
x_m	0.4574240
s_i	1.913141
k_1	3.869413
f_r	7.741895
L	367.9
J_0	1.286×10^{-3}

```
POP:=generation of the initial population
   REPEAT
(f_min, V_min):=best point among parent population
   (phases of the genetic reproduction)
         Selection
         Crossover
         Mutation
  (intensification)
     V_0 :=best n+1 individuals in population
     REPEAT
        Geometrical
              moves(reflection,
              expansion, contraction
                and multi-contraction)
     UNTIL stopping criteria are reached
     REPLACEMENT(the n+1 worst individuals in POP, the results of simplex)
    UNTIL stopping criteria are reached
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Fig. 2 Pseudocode of the hybrid method.

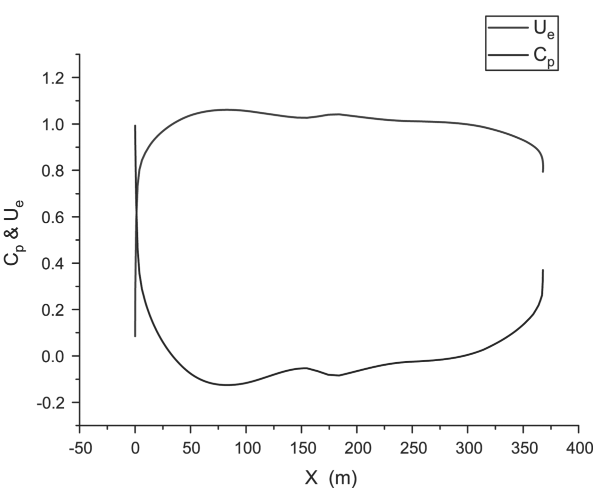

Fig. 3 Inviscid pressure coefficient and velocity distributions of optimization body.

of airship body in practical conditions. The optimization shape of real conditions can be gotten by considering the parameters bounds and others constraints using the method proposed in this Note. We have shown the optimization shape of airship body about one case in this Note:

$$r_n \in [0, 1.0], \qquad x_m \in [0.1, 0.9], \qquad s_i \in [0.001, 5.0]$$

$$k_1 \in [0.0, 10.0], \qquad f_r \in [1.5, 8.0]$$

During the course of computation, the number of points in a Reynolds-number regime $N = 10$, and weighting factors $\omega_i (i = 1, \ldots, 10) = 1.0$, the volume of airship body is 400,000 m^3.

The optimal shapes and pressure coefficient and velocity distributions of the case are shown in Figs. 3 and 4, respectively. The results of shape optimization of airship body are shown in Table 1.

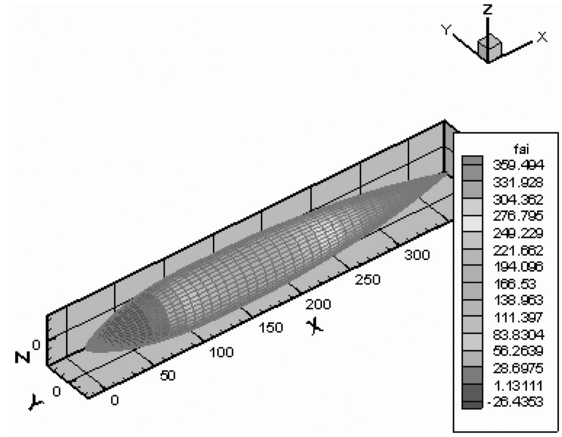

Fig. 4 Velocity potential distribution of optimization body.

VII. Conclusions

In this Note, we use the panel method to solve the Laplace equation. Because of the asymmetry of airship body, it can reduce the number of unknown parameters and increase the efficiency of computation through meshing the contour of airship along the x direction of body coordinates and circumference direction. The streamline contour can be denoted by the piecewise polynomials of eight parameters. Moreover, the contour of the optimization results[1,2] can be denoted by this method. So, we use the piecewise polynomials of eight parameters to denote the contour of airship in this Note. Using this method, every parameter has explicit domain. The number of optimal parameters is nine: eight polynomials parameters and the length of airship body. Using the source strengths and the segment lengths as optimal parameters,[1,2] the number of optimal parameters is much more than nine; moreover, the ranges of these parameters are implicit, which is bad for optimization.

The optimization of airship body is generated under constraint of given volume. During the course of optimization,[1,2] the constraint of given volume is satisfied by the approach of penalty function, which is a disadvantage. The constraint of given volume can be deleted by transforming it into the condition of a parameter satisfied by the method proposed in this Note. Moreover, the optimal parameters can be reduced from nine to eight. It is very good for the optimization of airship body.

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